Homework set #5 (assigned 14 April, due 23 April)

General comments. There is a lot of cool stuff in these problems and when you are done you will have derived some fairly sophisticated understanding of planets and the Solar System based on relatively simple ideas. All of these problems represent the techniques and approaches that are currently the state-of-the-art in planetary science.

There are fewer problems on this homework than on the others. Two of these problems are relatively straightforward math-y problems, and the third is the most involved problem I've given you yet. This is an opportunity to really stretch yourself and see how well you can do.

I suggest you read the entire problem set through before starting any of it. I suggest you start early so that you can ask me questions if you get stuck somewhere. You might need to think about some problems two or three times before the best approach becomes evident. Lastly, note that the symbol $\odot$ refers to the Sun, so $M_\odot$ is the mass of the Sun. I also remind you that an AU is an astronomical unit and that 1 AU is equal to 1.5×10^{11} m.

1) It has been suggested that the most or all of the Earth's water (around 10^{18} m^3) is not primordial (i.e., from the time of formation) but rather was brought in later by comets. Take a typical comet: 50% water ice, 10 km radius, density around 2000 kg/m^3. How many of these typical comets are needed to bring in the Earth's entire water supply of 10^{21} kg? If this massive influx of comets occurred during Late Heavy Bombardment (from around 4.5–3.9 billion years ago), what was the cometary influx rate (comets/year)? How many years go by between comet impacts? Compare this to the current cometary influx rate, which is something like 10^{-8} comets per year. How many years go by between comet impacts with the modern cometary influx rate?

Comets may contain, in addition to water ice, some amino acids. What is the total mass of amino acids that could have been deposited on Earth during Late Heavy Bombardment if we assume the above scenario is correct? You can assume that comets are 0.0003% (by mass) amino acids. We know that a pile of 6×10^{23} molecules of glycine (a typical amino acid) has a total mass of 75 g and that an amino acid might have a size around 10^{-9} m. Spread uniformly over the surface of the Earth, how thick is this extraterrestrially-derived amino acid layer? Are we talking about a thick or thin layer of imported amino acids? Discuss briefly (a few sentences).

2) The Initial Mass Function (IMF) describes how many stars of a given mass form. One theory about the IMF suggests that smaller mass stars are more likely to form than larger mass stars in this way: for 1 star with a given mass, there are 2.3 stars with one tenth that mass. If you assume that the nearest 100 stars have nice, friendly masses of either 10, 1, or 0.1 solar masses, how many of each mass are there in the nearest 100 stars? What percentage of the total mass does each group have?

If some wise, distant alien (let's say 1000 parsecs away) were observing us and these nearby stars, what would be the total luminosity of these 100 stars, and what percentage of the total luminosity would each of the three groups contribute?

You could make a table for this problem which lists all these things for the three groups of stars. You can leave your answers in terms of solar masses and solar luminosities. You should discuss your answer briefly.

For the purposes of this problem, you can assume that the following relationship for an individual star is true:

$$\left(\frac{L}{L_\odot}\right) = \left(\frac{M}{M_\odot}\right)^3. \tag{1}$$

3) We have talked about habitable zones in class. It is actually pretty straightforward to calculate the location of the habitable zone, both as a function of stellar temperature (that is, for different *stellar spectral types*) and distance, and as a function of time in the Solar System (remember the faint early Sun paradox and the continuous habitable zone).

The temperature of a planet (T_{pl}) is given by the following:

$$T_{pl}^4 = \frac{T_\star^4 R_\star^2 (1-A)}{4a^2} \tag{2}$$

where $T_\star$ is the temperature of the star heating the planet, $R_\star$ is the radius of the star, A is the albedo (reflectivity) of the planet, and a is the orbital distance of the planet.

Calculate the location of the habitable zone as a function of stellar temperature and distance. You should be able to reproduce the figure we saw in lecture. However, of course I want you to actually do the calculations and not just copy the figure from the lecture. Make a plot (stellar temperature versus distance from the star) that shows the location of the habitable zone. Mark the stellar spectral types on your plot as well. You can assume a planetary albedo of 0.5 (that is, 50% reflective; the Earth's albedo is more like 28%, but that's okay). You can also assume that stellar radii and stellar temperatures are related in this way:

$$\left(\frac{R}{R_\odot}\right) = \left(\frac{T}{T_\odot}\right)^{1.3} \tag{3}$$

which is not really exactly true but is good enough for us to use in this problem ($R_\odot$ and $T_\odot$ are the radius and temperature of the Sun). You should also ignore all atmospheric effects. This is obviously a terrible assumption, as a planet in the habitable zone with no atmosphere isn't going to be much of a habitable planet, but it makes the problem much easier.

You can also calculate the evolution of the location of the habitable zone in our Solar System over time. The luminosity of the Sun as a function of time is given by this (Gough 1981):

$$L_\odot(t) = \left[1 + \frac{2}{5}\left(1 - \frac{t}{t_\odot}\right)\right]^{-1} \times (L_{\odot,today}) \tag{4}$$

where t is time (in billions of years) and $t_\odot$ is the current age of the Sun, 4.5 billion years; and $L_{\odot,today}$ is the luminosity of the Sun today (3.9×10^{26} J/sec). Using this equation, make a plot (distance versus time) that shows the location of the habitable zone in our Solar System over time.

You might also find it useful to know that the luminosity of a star – the amount of energy the star emits per second – is given by this equation:

$$L_\star = 4\pi R_\star^2 \sigma T_\star^4 \tag{5}$$

where $L_\star$ is the luminosity (units: Joules per second); $R_\star$ is the radius of the star (units: meters); and $T_\star$ is the temperature of the star (in Kelvins). Sigma (σ) is the Stefan-Boltzmann constant (5.67×10^{-8} J/m^2/K^4/second).

On your plot showing the evolution of our Solar System's habitable zone with time, mark (shade in) the location of the *continuous habitable zone*. Also mark the Earth. Briefly (one or two sentences) comment. How would including a planetary atmosphere affect your answer?